Professor Solon I. Bailey, Harvard College Observatory, Cambridge, Mass., U.S.A.;
M. Charles Fabry, Professor of Physics in the University, Marseilles; and
Dr. W. H. Wright, Astronomer at the Lick Observatory, Mount Hamilton, California, U.S.A.,

were proposed by the Council as Associates of the Society.

Fifty presents were announced as having been received since the last Meeting, and the thanks of the Society were returned to the respective donors.

Proposal for a new Method of Determining the Constant of Aberration. By W. de Sitter, Ph.D.

1. Let four stars, or fields of stars, be selected in the equator at intervals of six hours. Let them be called I., II., III., IV., and let their right ascensions be α_1, α_2, α_3, α_4. At the epoch when I. is in the meridian at midnight the two arcs I.–IV. and II.–I. can be measured; the first or western angle (W) before midnight, and the eastern angle (O) after midnight. The difference of the two angles will be affected by aberration. Neglecting for the moment the influence of refraction, proper motion, etc., we have:

$$\begin{aligned}
\text{At epoch I.}: (\mathrm{W}-\mathrm{O})_1 &= (\text{I.–IV.}) - (\text{II.– I.}) + \mathrm{K}_1.\kappa,\\
\text{,, ,, II.}: (\mathrm{W}-\mathrm{O})_2 &= (\text{II.– I.}) - (\text{III.– II.}) + \mathrm{K}_2.\kappa,\\
\text{,, ,, III.}: (\mathrm{W}-\mathrm{O})_3 &= (\text{III.– II.}) - (\text{IV.–III.}) + \mathrm{K}_3.\kappa,\\
\text{,, ,, IV.}: (\mathrm{W}-\mathrm{O})_4 &= (\text{IV.–III.}) - (\text{I.–IV.}) + \mathrm{K}_4.\kappa,
\end{aligned}$$

where κ is the constant of aberration. Adding, we find:

$$\Sigma(\mathrm{W}-\mathrm{O}) = \Sigma\mathrm{K}.\kappa.$$

Calling now A_1 and D_1 the right ascension and declination of the point on the ecliptic whose longitude is 90° behind the sun at the epoch I., we have

$$\mathrm{K}_1 = 2\cos\mathrm{D}_1\left[\sin(\mathrm{A}_1-\alpha_1) - \cos\frac{\alpha_4-\alpha_2}{2}\sin\left(\mathrm{A}_1+\frac{\alpha_4+\alpha_2}{2}\right)\right].$$

The ideal case is $\alpha_4-\alpha_2 = 180°$, $\alpha_4+\alpha_2 = 2\alpha_1$, and $\mathrm{A}_1-\alpha_1 = 90°$. If these conditions were exactly fulfilled the factor within the square brackets would be unity. For differences of 1° in the arc $\alpha_4-\alpha_2$ and a fortnight in the epoch the factor is only diminished to 0·96. We can thus on the average take $\mathrm{K} = 1{\cdot}96\cos\mathrm{D}$. Now neglecting $\sin^6\epsilon$, we have

$$\Sigma\cos\mathrm{D} = 4 - \sin^2\epsilon - \tfrac{1}{4}\sin^4\epsilon(1+\tfrac{3}{2}\sin^2 2\alpha),$$

where α is any of the right ascensions $\alpha_1 \ldots \alpha_4$. The effect of the choice of the areas is thus very small: $2\Sigma \cos D$ varies between 7·65 and 7·67. We can thus reasonably expect a factor

$$\Sigma K = 7\cdot 5.$$

2. *Parallax* is rigorously eliminated if we observe at the exact epochs, and only a negligible effect can remain if the actual epochs are not exactly the best ones. There is thus no objection to the use of bright stars with unknown parallaxes.

Proper motion is not eliminated. We easily find

$$\Sigma(W - O) = \mu_4 - \mu_1.$$

To eliminate the effect of proper motion we must combine four series,* one beginning at epoch I., the next at epoch II., and so on, the observations being continued throughout seven epochs, or one year and a half. In other words, the several epochs must not have the same weight, but the relative weights must be:

At Epoch	I.	. . .	weight	1
,, ,,	II.	. . .	,,	2
,, ,,	III.	. . .	,,	3
,, ,,	IV.	. . .	,,	4
,, ,,	V. =	I. + 1 year	,,	3
,, ,,	VI. =	II. + 1 ,,	,,	2
,, ,,	VII. =	III. + 1 ,,	,,	1

Other corrections—nutation, diurnal aberration, aberration due to the eccentricity and the perturbations of the earth's orbit—should of course be applied. Refraction will be treated separately below.

3. The principal advantage claimed for the present method is its strictly differential character. The whole determination rests on the difference of two nearly equal arcs, which are measured on the same night with a few hours' interval. To fix ideas, let us suppose the observations to be made by Professor Turner's device for measuring arcs of 90°, explained in *M.N.*, vol. lxxi. Referring to fig. 9 on p. 428 of the volume quoted, we see that as the points

* We might derive the difference of proper motions $\mu_4 - \mu_1$ from the comparison of the arc I.–IV. as measured at the two epochs I. and IV. But we would then have to rely on the constancy of the instrumental arc of 90° for nine months, and the differential character of the method would be lost. Moreover, the loss of weight would be not inconsiderable. The mean error of $\mu_4 - \mu_1$ thus determined would be $\pm\frac{4}{3}\sqrt{2}.\epsilon$ (for the meaning of ϵ, see art. 4 below), and the resulting mean error of κ from one series would be increased from $\pm 0\cdot 370\,\epsilon$ to $\pm 0\cdot 435\,\epsilon$.

The constancy of the angle can, of course, be verified by Turner's cube (*M.N.*, lxxi. p. 428, fig. 8). It must be left to practical considerations to weigh the disadvantages connected with this introduction of another instrumental adjustment, and with the loss of weight, against the difficulty of securing the necessary number of observations at the middle epochs, which must have the larger weights.

S_1 and S_2 in their daily motion describe the equator from east to west, the points T_1 and T_2 describe the horizon from west to east, always conserving their distance of 90°. The mirror M remains fixed. The stability of this mirror is, however, only of secondary importance. A slight displacement would throw the stars out of the field, but would have no effect on the measured arc. The only quantity which must be trusted to remain constant during one night is the angle between the collimation lines of the two telescopes. This angle need not be known; it need not be exactly 90° [indeed it may be advantageous to make it 89° 58′ to allow for refraction]; it must only be constant. It would, of course, be possible to measure the arcs of 90° by any other instrument, such as M. Bigourdan's "*Comparateur*," described in *Comptes Rendus* for 1915 January 25, which is just coming to

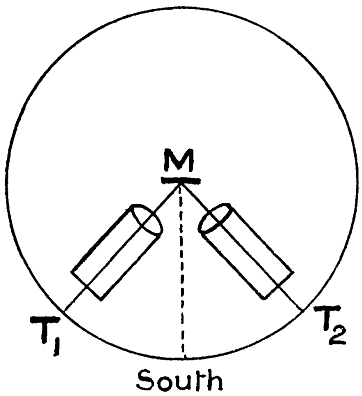

FIG. 1.

hand as this paper is being prepared for the press. But Turner's method seems to give the best chances of stability.

The invariability of the arc T_1 T_2 is of course best secured if the two telescopes are permanently fixed in azimuths + and − 45°, as shown in the figure.

But suppose them to be moved in azimuth without changing their angle, then we find the following zenith distances ζ_1, ζ_2, and angles of incidence N_1, N_2 for the two stars S_1, S_2 for different hour angles τ of the middle point (the table has been computed for latitude $\phi = 50°$):

τ	ζ_1	ζ_2	N_1	N_2
h m	°	°	°	°
0 0	63·0	63·0	48·4	48·4
0 30	67·0	59·3	55·1	41·8
1 0	71·3	56·2	62·0	35·5
1 30	75·8	53·6	68·9	29·8
2 0	80·4	51·6	75·9	24·8

Thus if we wish to avoid large zenith distances, and large angles of incidence, we are restricted to small hour angles τ. The

interval between the two observations on the same night is 6 hours $- 2\tau$. On the whole, it seems best to take $\tau = 0$.

4. Let the mean error of one measured arc of 90° be ϵ. Then, taking one observation at each epoch, the mean error of $\Sigma(W - O)$ is $\pm 2\sqrt{2}.\epsilon$. Thus from one series the mean error of κ would be, taking no account of the effect of proper motion, $\pm \frac{2\sqrt{2}}{7 \cdot 5}\epsilon$, and from four series so combined as to eliminate proper motion, one half of this, or $\pm 0 \cdot 185\epsilon$.

The only other method hitherto used for the determination of the constant of aberration, which also depends on differences between two observations on the same night, is Küstner's [*Astr. Nachr.*, Band 126]. In Küstner's method we measure differences of zenith distances. If we take four stars (or groups of stars), the observations are made at four epochs, and two observations on each night, with intervals of six hours. Thus if we take one observation of zenith distance as equivalent to one measure of an arc of 90°, the number of observations for the two methods is exactly the same. In Küstner's method we must trust to the stability of the instrumental zenith, which is controlled by observations of the spirit-level. It is evident that the uncertainty introduced thereby is of quite a different order from the possible want of constancy of the angle between two fixed horizontal telescopes.

Further, in Küstner's method proper motion is not eliminated. The method here proposed is thus differential in a much higher degree than Küstner's.

Moreover, the factor is about twice as large. For Küstner's method, taking four stars of declination $\delta = \phi$, we find for the factor of κ in one difference of zenith distance,

$$K = 2 \sin\phi \cos D \sin\frac{\alpha_1 - \alpha_2}{2} \sin\left(A - \frac{\alpha_1 + \alpha_2}{2}\right).$$

Taking the exact epoch $A - \frac{\alpha_1 + \alpha_2}{2} = 90°$, and also $\alpha_1 - \alpha_2 = 90°$, we find, for $\phi = 50°$,

$$K = 1 \cdot 083 \cos D,$$

as against $K = 2 \cos D$ for the method here proposed. Therefore, other things being equal [which they are not], with Küstner's method we require four times the number of observations to get the same mean error in the result.

Küstner's method, on the other hand, has the advantage of working near the zenith, and thus avoiding refraction. It is thus important to consider whether the advantages of the present method are not neutralised or outweighed by the uncertainty introduced by the correction for refraction.

5. *Refraction.*—The effect of refraction on the right ascension of an equatorial star is $k \tan t$, where t is the hour angle. We have,

for the two stars making an angle of $90°$, $t_1 + t_2 = 90°$, $t_1 - t_2 = 2\tau$. Therefore the effect on the measured arc W or O is

$$2k \sec 2\tau.$$

The two arcs W and O are measured at the same hour angle τ. The effect of refraction on the difference W − O is therefore

$$2(k_w - k_o) \sec 2\tau.$$

As has been explained above, the best plan is to have $\tau = 0$, $\sec 2\tau = 1$. The difference $k_w - k_o$ depends on the temperature and the barometric pressure. Approximately we have, if the temperature is measured in degrees Centigrade and the pressure in millimetres:

$$2(k_w - k_o) = 0''\cdot43(T_o - T_w) + 0''\cdot15(B_w - B_o).$$

It is difficult to estimate the accuracy of the difference in refraction derived by the usual formulas from the readings of the barometer and the thermometer. Suppose we determine the difference of temperature $T_w - T_o$ with a mean error of $\pm 0°\cdot1$, and $B_w - B_o$ with a mean error of $0^{mm}.1$. The assumption made is not that we are able to read our meteorological instruments with this accuracy, but that the real difference of refraction is identical with the difference derived from our formulæ within limits of uncertainty corresponding to the stated mean errors of $T_w - T_o$ and $B_w - B_o$ respectively. Then the uncertainty in one measured value of W − O will be characterised by a mean error of $\pm 0''\cdot045$. If we take only *four* nights at each epoch, the corresponding mean error in $\Sigma(W - O)$ will again be $\pm 0''\cdot045$, and in $\kappa \pm 0''\cdot006$. If four series are combined so as to eliminate proper motion, the uncertainty in the resulting value of the constant of aberration due to refraction will be $\pm 0''\cdot003$.

6. *Alternative Method.*—If we measure azimuths instead of right ascensions, we get rid of refraction altogether.

We have $\tan A = \tan t \operatorname{cosec} \phi$, therefore, for $\phi = 50°$ and $t = 45°$, we have $A = 52°\cdot5$; the difference of azimuth of two consecutive areas when the middle point is on the meridian is therefore $105°$. Now suppose two fixed horizontal telescopes making this angle symmetrically to the meridian, each with its own mirror.

The normals to the mirrors must be in the vertical planes passing through the optical axes of the telescopes, and inclined $45° - \frac{1}{2}\zeta$ to the horizon. For $\phi = 50°$ this becomes $13°\cdot5$. The angles of incidence are thus small. This is an advantage. Each area will be reflected by its own mirror into its own telescope. The mirrors must be fixed. The inclination of their normals is of secondary importance, but their orientation, *i.e.* the adjustment of their normals in the vertical planes through the optical axes of the telescopes, must be supposed to remain invariable during one night. This will not be difficult to realise.

The observations must be accurately timed, as much as possible

at the exact epoch $\tau = 0$. The observation is changed in character and becomes practically an observation of the (oblique) transits of the stars through the vertical planes of collimation of the two telescopes. It is evident that higher claims are made on instrumental stability than with the simple method of one mirror. Whether these disadvantages are outweighed by the advantage

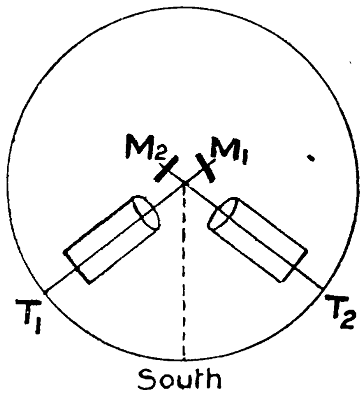

FIG. 2.

of the elimination of refraction must be left to experiment to decide.

Parallax and proper motion are eliminated as before. The factor now becomes (for the ideal case of arcs of exactly 90° and observations at the exact epochs):

$$K = 2 \operatorname{cosec} \zeta \sin (q - D),$$

where q is determined by

$$\sin q = \cos A \sec t, \qquad \text{or} \qquad \tan q = \tan \phi \operatorname{cosec} t.$$

In $\Sigma(W - O)$ we have thus

$$\Sigma K = 2 \operatorname{cosec} \zeta [\sin q \Sigma \cos D - \cos q \Sigma \sin D].$$

Now $\Sigma \sin D$ is zero, and consequently the factor for the original method is multiplied by $\sin q \operatorname{cosec} \zeta$. For $\phi = 50°$ this is 0·965. The factor thus becomes 7·14 instead of 7·5. If the epochs are not exact, we have, still taking arcs of exactly 90°,

$$K = 2 \operatorname{cosec} \zeta [\sin (A_1 - \alpha_1) \cos D_1 \sin q - \sin D_1 \cos q].$$

The loss of factor due to deviation of the epoch from the exact date is thus also of the same order as in the method of right ascensions.

7. The word "new" in the title of this note is not meant to imply the hypothesis that the idea of determining the constant of aberration from measures of large arcs, or even from differences of such arcs, has never occurred to anybody besides the present author. It is, of course, at least as old as 1871. But the cyclical arrangement of the four equatorial arcs has never been used and, so far as

known to me, never been expressly proposed for the determination of the constant of aberration.

Professor Turner, when discussing his scheme of subdivision of the equator by arcs of 90° for the purpose of determining fundamental right ascensions, necessarily was led to consider the question of the correction of the observations for aberration. Incidentally, he mentions the possibility of determining the value of the constant from the differences of adjacent arcs.* But nothing is said about the arrangement of the observations or the choice of the arcs, and it is not clear from the quoted passage whether Turner (from whose point of view the constant of aberration was not, of course, the primary object of investigation) did realise the full capacity of the method as a means of determining this constant.

The well-known method of Loewy, which was developed in a series of communications to the Académie des Sciences in the course of the year 1887 (*Comptes Rendus*, vol. civ.), depends on the comparison of *two* arcs, differently affected—preferably one of them not affected—by aberration, at two epochs. The method is thus very different from the one proposed above. Theoretically it is differential, in so far as the lengths of the arcs and the variation of the angle of the prism between the two epochs (not between the two observations at the same epoch, which are nearly simultaneous) are eliminated from the final result. But the completeness of this elimination must depend on the extent to which the instrument can be trusted to be invariable under the conditions in which the observations are made. The two arcs are observed in widely different azimuths. The optical and mechanical stability of the equatorial telescope, with its prism mounted before the object-glass, is naturally very different from that of a system consisting of two fixed horizontal telescopes and a fixed mirror. The fact that the two areas at both ends of one arc are viewed through different parts of the object-glass introduces a new source of errors. It may be noted that theoretically the maximum factor—for the same number of observations—is the same as in the method of the four equatorial arcs; but if day observations are to be avoided, it is not possible to observe at the exact epoch, and according to Loewy's estimate not more than $\frac{3}{4}$ of the maximum factor can be obtained in practice and not more than $\frac{1}{2}$ in an extended series of observations (*l.c.*, p. 1654). For the same reason proper motion can only very imperfectly be eliminated. Parallax is not eliminated at all. For all these reasons Loewy's method appears to me to be considerably inferior to the method proposed in the present paper.

* "Now we can compare any particular pair of arcs at times of year differing by a few months, . . . and in this way we may get a sufficiently strong determination of aberration which can be used for correcting the comparisons." *Monthly Notices*, lxxi. p. 434.

Leiden:
1915 *February.*

A Proposal for the Comparison of the Magnitude Scales of the Astrographic Catalogue. Sixth Note. The Oxford Magnitudes. With a Preliminary Discussion of the Existence of Obscured Patches in the Sky. By H. H. Turner, D.Sc., F.R.S., Savilian Professor. *With a letter from* F. G. Brown.

1. The previous notes are in *Monthly Notices*, lxix. p. 392, lxxii. p. 464 and p. 700, lxxv. p. 57 and p. 143. The proposal is

That the number of images recorded under each unit of the magnitude scale be counted and tabulated.

Complete zones of the Bordeaux, Algiers, Cape, and Perth (W.A.) portions have been given as examples, and two zones $+64°$ and $+62°$ of the Vatican results have been counted: but since these last counts are not very accordant, it is considered better to count yet another zone before presenting the figures. The counts for the Oxford zones will be given in full detail in vol. viii. of the Oxford Astrographic Catalogue, so that no extended tables need be given here: but it is desirable to have for comparison with other zones the figures of Table I.

2. In Table I. are given a few counts from the middle zone $(+28°)$ of the seven $(+25°$ to $+31°)$ which form the Oxford share. The numbers of plates in 0^h, 1^h, and 2^h are 7, 7, 6, and this triplet repeats for the other octants of R.A. It has not been considered necessary here to reduce these figures to uniformity. The counts have actually been made to every unit of recorded diameter (d) of the star image: a selection is made here for brevity, but the counts for the fainter stars are given in some detail, in order to exhibit the vagaries of the scale near the limit of faintness —partly due to idiosyncrasies of the measurers, which will be fully investigated. But here as elsewhere it is considered that the total number of stars on the plate is steadily consistent. For a few plates the measurement was limited to the brighter stars to save labour: but the number of stars omitted was counted and printed in the volumes, and the requisite additions are made in the column "All."

[Table.